

NATIONAL UNIVERSITY OF LESOTHO

B.A. SUPPLEMENTARY EXAMINATION

EC3303: PRINCIPLES OF MATHEMATICAL ECONOMICS

August 2023

100 Marks

Time: 3 hours

INSTRUCTION: Answer any FOUR questions

Question 1

- a) Suppose your exogenous income is R200, the price of shirts is R10 and the price of pants is R20. Suppose further that, you held a coupon that gives you 50% off the first 6 pair of pants (when the regular price of pants is R20.) What will be your choice set? (4)
- b) Sometimes, money that can be devoted to consumption is not *exogenous*, but arises *endogenously* from the decisions a consumer makes and the price vector she faces in the market. If a consumer returned to the shop with 10 shirts and 5 pants to get a store credit at the current price, his income can be expressed as $I = 5p_1 + 10p_2$. What will be his choice set? (2)
- c) The quantity of a commodity supplied to the market when the selling price is P takes the form of:

$$Q = a + bP + cP^2, \text{ for some constants } a, b \text{ and } c.$$

It is known that when $P = 2$, the quantity supplied is $Q = 7$; when $P = 3$, the quantity supplied is $Q = 16$ and when $P = 4$, the quantity supplied is $Q = 29$.

- i) Formulate a system of three linear equations in the unknowns of a, b and c . (3)
- ii) Express the system of linear equations in matrix form and solve it. (13)
- iii) What would be the quantity supplied if the price was $P = 1$? (3)

Question 2

- a) Find the eigenvalues and their corresponding eigenvectors of matrix $A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$ (6)
- b) Find the rank of the matrix $B = \begin{bmatrix} 0 & -11 & -4 \\ 2 & 6 & 2 \\ 4 & 1 & 0 \end{bmatrix}$ (4)
- c) Determine the sign definiteness of the function $5x_1^2 + 8x_1x_2 + 5x_2^2$. (5)
- d) Diagonalize matrix $C = \begin{bmatrix} -3 & -4 \\ 5 & 6 \end{bmatrix}$ (5)
- e) The Utility function of a hypothetical household consuming commodity x is given by

$$U(x) = \frac{x^2}{(x-4)}, \text{ locate the position and the nature of any turning points on this function. (5)}$$

Question 3

a) In each of the following cases, determine whether the function is homogeneous or not. If it is so, find the degree of homogeneity?

i) $x^2y + 6x^3 + 7$ (2)

ii) $g(x, y, z) = \frac{\sqrt{3x^2+5y^2+z^2}}{4x+7y}$ (3)

b) Verify Euler's Theorem for the function $u = \frac{1}{\sqrt{x^2+y^2}}$ (4)

c) Show that the Cobb-Douglas production function is homogeneous and it satisfies the Euler's theorem. (4)

d) If $Q = 32K^{0.5}L^{0.4}$ and $K = 0.2L^2$, what is the total change in L and Q? Identify the direct and indirect effects. (4)

e) If utility function is given by $U = f(x, y) = 8x^2 - 5xy + 6y^2$, where $x = g(y) = \frac{1}{y}$.

Find the total change in commodity y on the utility function of the consumer. (4)

f) Find the derivative of the function $x^3 + x^2y + 4y^2 = 6$ (4)

Question 4

a) What is the sign definiteness of the quadratic function $-x_1^2 - x_2^2$? (4)

b) Suppose that both the of capital at time t, $K=K(t)$, and that the efficiency with which it is used affect GNP according to the function $y = f(k, t) = 0.2(1 + t)^{1/2}k$ where $k = k_0e^{0.05t}$

Find and give the economic intuition of the derivative:

$$\frac{dy}{dt} = f_t + f_k \frac{dk}{dt} \quad (8)$$

c) Find the derivative of $x^2y^3 + 3xy^2 + y = 22$ (5)

d) Find the stationary values of the following functions and use second -order conditions to determine which give maxima, minima or saddle points:

i) $y = x_1 + x_2 - x_1^2 - x_2^2 + x_1x_2$ (6)

ii) $y = 2x_1x_2 - x_1^3 - x_2^2$ (6)

Question 5

a) The total revenue and total cost functions of a monopolist given by:

$$TR = 29Q - 3Q^2$$

$$TC = \frac{Q^3}{3} - 6Q^2 + 2Q + 40$$

i) Find the profit maximizing output. (6)

ii) What is the maximum profit? (2)

b) Minimize a firm's cost function $c(l, k) = 3l^2 + 5lk + 6k^2$, where l = labour and k = capital, when the firm must meet a production quota of $5l + 7k = 732$ by:

i) Finding the critical values through Cramer's rule. (8)

ii) Using the Bordered Hessian to test for the second-order conditions. (4)

c) Consider the demand equation: $p = -0.5x + 50$ ($0 \leq x \leq 100$), which describes the relationship between the unit price p in Maloti and the quantity demanded x of noodles.

Find the elasticity of demand $E(p)$ at $p = 25$. (5)

Question 6

The constant elasticity of substitution (CES) production function is given by

$$F(K, L) = (aK^{-\beta} + bL^{-\beta})^{-1/\beta} \quad (a > 0, b > 0) \text{ where } a, b \text{ and } \beta \text{ are constants}$$

a) Show that it is homogeneous of degree 1. (5)

b) Is the marginal product of capital a homogeneous function? If homogeneous, what is its degree of homogeneity? (7)

- c)** Verify that the CES production function satisfies Euler's theorem. **(8)**
- d)** Calculate the elasticity of substitution for the CES production function. **(5)**