**NATIONAL UNIVERSITY OF LESOTHO**

**B.A. EXAMINATIONS**

**EC3303: PRINCIPLES OF MATHEMATICAL ECONOMICS**

**January 2024** **100 Marks** **Time: 3 hours**

**INSTRUCTION:** Answer any FOUR questions

**Question 1**

**a)** A monopolist offers two different products, each having the following market demand functions:

$$q_1 = 14 - \frac{1}{4}p_1$$

$$q_2 = 24 - \frac{1}{2}p_2$$

The monopolist's joint cost function is

$$C(q_1, q_2) = q_1^2 + 5q_1q_2 + q_2^2$$

**i)** Find the critical value of $q_{1,}q_{2,}p_1 \ and \ p_2$ and calculate maximum profit. **(16)**

**ii)** Verify that the profit is at maximum. **(4)**

**b)** Compute the first and second derivatives of the following function:

$$y = 4 + 4x^{3/4}$$

From your results, state whether the function is increasing/decreasing, the rate at which it is increasing/decreasing and the curvature of the function. **(5)**

**Question 2**

**a)** A thousand people took part in a survey to reveal which of these newspaper, A, B or C, they had read on a certain day. The responses showed that 420 had read A, 316 had read B, and 160 had read C. These figures include 116 who had read both A and B, 100 who had read A and C, and 30 who had read B and C. Finally, all these figures include 16 who had read all three papers.

**i)** How many had read A, but not B? **(3)**

**ii)** How many had read C, But neither A nor B? **(3)**

**iii)** How many had read neither A, B, nor C? ` **(3)**

**iv)** Denote the cardinality of the complete set of all people in the survey. **(3)**

**b)** State and prove these two laws:

**i)** The complement of the union of any family of sets equals the intersection of all set's complements. **(4)**

**ii)** The complement of the intersection of any family of sets equals the union of all set's complements. **(4)**

**c)** Given the set $S = \{1,2,3,4\}$, define all its possible subsets. How many are they? **(5)**

**Question 3**

**a)** Use matrix inverse to obtain Y, C, and I for the following model: **(15)**

$$Y = C + I + G$$

$$C = 100 + 0.6Y$$

$$I = 50 + 0.2Y$$

$$G = 500$$

**b)** Use the eigenvalues to determine the sign definiteness of the quadratic form of

$B = 3x^2 + 2xy - y^2$ . **(7)**

**c)** Write the following as $X^T AX$.

$A = 5x_1^2 + 3x_2^2 + 2x_3^2 - x_1x_2 + 8x_1x_3$ **(3)**

**Question 4**

**a)** Find the rate of change of output with respect to time, if the production function is

$Q = A(t)K^\alpha L^\beta$ , where $A(t)$ is an increasing function of $t$,

$K = K_0 + at \;\; and \;\; L = L_0 + bt$ **(8)**

**b)** According to a study, the demand quantity Q for butter in Stockholm during the period 1925-1937 was related to the price $P$ by the equation $QP^{1/2} = 38$.

Find ${dQ}/{dP}$ by implicit differentiation and check the answer by using different method to compute the derivative. **(7)**

**c)** Write Taylor series expansion for $f(x) = \frac{1}{1+x}$, around $x_0 = 1 \;\; with\, n = 4$. **(5)**

**d)** Consider the demand equation:

$p = -0.5x + 50 \quad (0 \leq x \leq 100)$, which describes the relationship between the unit price $p$ in Maloti and the quantity demanded $x$ of noodles.

Find the elasticity of demand $E(p)$ at $p = 25$. **(5)**

**Question 5**

**a)** For the function given below:

$$Q = AK^{\alpha}L^{1-\alpha}$$

**i)** Express $MP_L$ as a function of $\frac{K}{L}$ ratio. **(7)**

**ii)** Show that $AP_L$ is the function of $\frac{K}{L}$ ratio. **(3)**

**b)** Minimize a firm's cost function $c(l,k) = 3l^2 + 5lk + 6k^2$, where $l =$ labour $and$ $k =$ capital, when the firm must meet a production quota of $5l + 7k = 732$ by:

**i)** Finding the critical values through Cramer's rule. **(12)**

**ii)** Using the Bordered Hessian to test for the second-order conditions. **(3)**

**Question 6**

**a)** In each of the following cases, determine whether the function is homogeneous or not. If it is homogeneous, find the degree?

**i)** $Z(x,y) = x^2y + 6x^3 + 7$ **(2)**

**ii)** $g(x,y,z) = \frac{\sqrt{3x^2+5y^2+z^2}}{4x+7y}$ **(3)**

**iii)** $h(x,y) = \frac{6x^2y^3-\pi y^5+9x^4y}{2020x^2+2019y^2}$ **(2)**

**b)** Verify Euler's Theorem for the function $u = \frac{x^2+y^2}{\sqrt{x+y}}$. **(6)**

**c)** The constant elasticity of substitution (CES) production function is given by

$F(K,L) = (aK^{-\beta} + bL^{-\beta})^{-1/\beta}$ $(a > 0,\ b > 0)$ where $a, b$ and $\beta$ are constants

**i)** Show that it is homogeneous of degree 1. **(5)**

**ii)** Is the marginal product of capital a homogeneous function? If homogeneous, what is its degree of homogeneity? **(7)**