NATIONAL UNIVERSITY OF LESOTHO

MSC. ECONOMICS EXAMINATIONS

EC6035 – ADVANCED MATHEMATICAL METHODS IN ECONOMIC ANALYSIS

January, 2024	**100 Marks**	**Time: 3 Hours**

***Instructions**:*

Answer any FOUR questions.

Question 1

a) Prove that the sequence $\{x_k\}$ with general term given by

$$x_k = \sum_{r=1}^{k} \frac{1}{(r+1)(3r+4)}$$

is a Cauchy sequence. **[10]**

b) The management of Standard Lesotho Bank is considering investing a liquid asset that has been dormant for about two years in either stocks A or stock B or both. They have intentionally kept this asset aside to meet unexpectedly high withdrawals from some customers. Assuming the expected annual return from stocks A and B are the same, you have been approached to offer advice on:

- the proportions of each asset to invest in each stock.
- the risk involved in the investment decision.

Suppose the risk involved in investing in stocks A and B (as measured by the standard deviation) are respectively 5% and 7%. What advice would you give in each of the following scenarios as a rational consultant?

i) If stock A does poorly, stock B does well (the correlation between stocks A and B is -0.9). **[9]**

ii) If stock A does well, stock B does well too (the correlation between stocks A and B is 0.9). **[6]**

Question 2

a) Given the set X with objects α, β, γ and δ, provide two topologies $\mathcal{T}_1$ and $\mathcal{T}_2$ which are topologies on X. Demonstrate that $\mathcal{T}_1$ and $\mathcal{T}_2$ are topologies on X. Your topologies should exclude the discrete and indiscrete topologies. **[10]**

b) A company has decided to produce new cereal called "Nuts and Bran" which contains only nuts and bran. It is to be sold in standard size pack which must contain at least 375g. To provide an "acceptable" nutritional balance, each pack should contain at least 200g of bran. To satisfy the marketing manager, at least 20% of the cereal's weight should come from nuts. The production manager has advised you that nuts will cost M20 per 100g and that bran will cost M8 per 100g.

i) Formulate a linear programming model for the minimization of cost. **[4]**

ii) Solve your linear programming model graphically. **[6]**

iii) Determine the cost of producing each packet of cereal. **[2]**

iv) Which constraints of your model are binding? Which constraints are nonbinding? **[3]**

Question 3

a) Verify that (1,0,0), (0,1,0), (0,0,1) and (0.267,0.534,0.8022) are unit vectors in $\mathbb{R}^3$. Which of these vectors are orthogonal? **[8]**

b) Suppose that a firm produce three outputs, y_1, y_2 and y_3 with three inputs z_1, z_2 and z_3. The input-requirement matrix is given by

$$A = \begin{pmatrix} 3 & 1 & 2 \\ 2 & 5 & 1 \\ 1 & 1 & 3 \end{pmatrix}$$

If the firm wants to produce 10 units of y_1, 20 units of y_2, and 10 units of y_3, how much of z_1, z_2 and z_3 will it require? **[6]**

c) Show that the function $y = f(x_1, x_2) = x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}},\ \ x_1, x_2 > 0$ is strictly concave. Hence demonstrate that the total differential overestimates the actual change in the function. Use the initial point $\mathbf{x} = (1,2)$ and changes in the x_i values $dx_1 = 3$ and $dx_2 = 7$. **[11]**

Question 4

a) Solve the following constrained maximization problem

$$\max\ (x_1^2 x_2)$$

subject to

$$x_1 + x_2 \leq 60$$

$$3x_1 + x_2 \leq 70$$

$$x_1 \geq 0, x_2 \geq 0$$

by the Kuhn-Tucker method indicating whether the stationary values are quasiconcave or quasiconvex. **[17]**

b) Suppose the government has been taxing each person's income at a marginal rate of 0.4 for every maloti in excess of M25,000, with the first M25,000 earned not taxed. In addition, the government imposed a lump-sum surtax of M2,000 on everyone who earns M100,000 or more. Write out and graph income tax after tax, y, as a function of income before tax, x. Indicate why, according to the definition of continuous function, the function has a point of discontinuity at $x = M100{,}000$. Discuss any incentive effects on hours worked that may arise due to this discontinuity in the tax schedule. **[8]**

Question 5

2. a) A monopolist supplies two markets, one at home, the other abroad. The demand functions are

$$q_1 = 10 - p_1,\; q_2 = 5 - 0.5p_2$$

where q_1 denotes the home sales, and q_2 foreign sales. p_1 and p_2 are the respective home and foreign prices for these markets. The firm's total-cost function is

$$C = 0.5(q_1 + q_2)^2$$

i) Find its profit-maximising output and prices (assuming that no arbitrage between the markets is possible). **[7]**

ii) Suppose now that there is price regulation that forbids the monopolist supplier from selling beyond the price b at the local market. Find the range of values of b for which a reduction in price gladdens the heart of both the monopolist supplier and home market purchasers. [Hint: express the profit-maximising function in terms of b and plot]. **[8]**

b) Find the limits of the following functions:

i) $f(x) = \lim_{x\to\infty} \frac{\exp(x^2)}{x^3}$ **[2.5]**

ii) $g_\lambda(x) = \lim_{\lambda\to 1} \frac{x^{1-\lambda}-1}{1-\lambda}$ for $\lambda \geq 1, x > 0$. **[2.5]**

iii) $h(x) = \lim_{x\to\infty} x^{\frac{1}{x}}$. **[2.5]**

iv) $k(x) = \lim_{x\to 0} \frac{sinx - x^2}{x^3}$. **[2.5]**